NEW UNAMBIGUOUS BOC(N,N) TRACKING TECHNIQUE

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INTRODUCTION

Galileo and Modernized GPS have included in their signal structures a new signal modulation: the Binary Offset Carrier (BOC). In the navigation field, this modulation is characterized by the chipping rate of its spreading code \((m \times 1.023 \text{ MHz})\) and the frequency of its square sub-carrier \((n \times 1.023 \text{ MHz})\). As a consequence, it is usually referred to as a BOC\((n,m)\) modulation. The choice of the parameters \(n\) and \(m\) has a significant impact on the signal tracking performance and characteristics \([1]\). It is well known that each BOC modulation brings many improvements when compared against a classical Bi-Phase Shift Keying (BPSK) modulation with the same chipping rate \([1]\). Among other examples, it provides a lower code tracking error in thermal noise, better multipath mitigation, and better rejection of narrow-band interference. However, its multi-peak autocorrelation function is a major drawback. This implies that when using classical acquisition and tracking techniques, there is a possibility of detecting and tracking the signal by locking onto a side-peak. This can lead to severe undesirable measurement biases when not corrected.

Several methods have been developed in order to prevent such an event to occur \([2, 3, 4, \text{ and } 5]\). These methods are usually generic to all BOC modulations. However, they have to incorporate trade-offs, such as a degraded code tracking accuracy, or the risk of a certain period of potential false peak tracking. This research uses a different approach to the problem. Instead of trying to find a generic solution, it was decided to study this ambiguity problem on a particular signal in order to try to apply relevant methods using this specific signal’s characteristics. The BOC modulation chosen was the sine-BOC\((n,n)\) (sine- stands for the use of a sine square as the sub-carrier). This decision was motivated by the fact a sine-BOC\((1,1)\) will most likely be used for the Galileo civil signal on L1, and potentially on GPS III \([6, 7]\). Moreover, the Galileo L1 signal will constitute the main Galileo signal for mass market applications due to its narrow frequency bandwidth and low sampling frequency required for its processing. As a result, finding an optimal way to track Galileo BOC\((1,1)\) unambiguously is critical, as well as very challenging.

This paper presents a new unambiguous BOC tracking method that can be applied to any sine-BOC\((n,n)\) signal. It consistently removes the bias threat while having a close-to-optimum tracking accuracy. Sine-BOC modulation will be referred to as BOC modulation for simplicity throughout this paper. The first part focuses on the shortcomings of traditional BOC\((n,n)\) tracking and the second part offers a detailed description of the new proposed method. The third part then focuses on the theoretical tracking performance of the proposed method in thermal noise only. Its behaviour in a multipath environment is then investigated before some simulation results are shown in the last section.

BOC(N,N) TRADITIONAL TRACKING

The theoretical expression of the BOC\((n,n)\) autocorrelation function, \(R_{\text{BOC}}\), is given by \([8]\):

\[
R_{\text{BOC}}(\tau) = tri_{\frac{1}{2}}\left(\frac{\tau}{1}\right) - \frac{1}{2} tri_{\frac{1}{2}}\left(\frac{\tau}{1}\right) - \frac{1}{2} tri_{\frac{1}{2}}\left(\frac{\tau}{1}\right)
\]
where \( \text{tri}\left( \frac{x}{y} \right) \) is the value in \( x \) of a triangular function centred in \( z \) with a base width of \( y \) and a peak magnitude of 1; all values are given in chips; and \( \tau \) is the code delay in chips.

From (1), it is easy to see that there are two side-peaks present on the BOC(n,n) autocorrelation function at \( \pm 0.5 \) chips. Their magnitude is very significant, as it reaches half of the main peak’s magnitude. This can create significant problems when a traditional acquisition scheme is applied, as it is based on energy detection. For high \( C/N_0 \), the magnitude of the three peaks will increase equally compared with the noise level, not offering a better isolation of the main peak against the side-peaks. For low \( C/N_0 \), the high post-correlation noise will induce a significant risk of false detection. This analysis has already been made in [8].

As far as tracking is concerned, studying the discriminator output is the best way to understand the false lock threat. Fig. 1 (Left) represents the output of a normalized Early-Minus-Late Power (EMLP) discriminator [8]. It is obvious that the discriminator output in such a case crosses zero ‘in the right direction’ for a null code delay, but as well for code delays of \( \pm 0.56 \) chips. Consequently, these two points can be considered as potential false lock points. Many events can trigger such a false lock such as an incorrect acquisition, a faulty transition from acquisition, a short loss of code lock, strong multipath, or high thermal noise. Fig. 1 (Right) represents the case when the acquisition process led to a -0.5 chip bias, followed by a false lock during tracking.

![Fig. 1 – Standard Normalized BOC(1,1) EMLP Discriminator for an Early-Late Spacing of 0.2 Chips (6 MHz Double-Sided Front-End Filter) (Left), and Example of Biased BOC(1,1) Tracking on False Peak with an Initial Code Delay Error of -0.5 Chips (2 Hz DLL) (Right)](image)

Previous methods introduced to eliminate such a potential bias threat have two main aspects. (1) A constant check of the magnitude of the peak currently tracked versus the side-peaks’ magnitudes; (2) the use of multiple correlators or a different local code in order to obtain a modified correlation function that is unambiguous.

The first type of solution, often referred to as the ‘Bump and Jump’ technique [2], uses Very Early (VE) and Very Late (VL) correlators situated on the side peaks. By doing so, it is possible to control if the receiver is tracking the correct peak, and if not, to make the code tracking loop jump onto the main peak. This method seems to work well down to fairly low \( C/N_0 \) values when only thermal noise is assumed [9]. However, the chances of not being biased are not zero, which might not be tolerable for certain critical applications. Moreover, there is a concern when both low \( C/N_0 \) and large multipath affect the correlation, as it can greatly influence the relative magnitude of the main and secondary peaks.

The second type of ambiguity cancellation method usually uses a correlation function whose main peak is wider than the original BOC autocorrelation function [3, 4, and 5], but unambiguous, often implying a sensible increase in the tracking error variance and reducing the resistance to multipath, such as the single side lobe method. However, it does not rely on a checking process that could be unreliable, and as a consequence it appears to be more robust. It is in this direction that the research presented in this paper focuses on, with the intent to keep a narrow correlation peak, as shown in the next section.

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The essence of the proposed technique is the desire to remove completely any side-peak from the BOC(n,n) autocorrelation function, or in other word, to remove the two last components of (1). This is possible when deriving the expression of the correlation function $R_{BOC/PRN}$ of the BOC(n,n) signal with its spreading code without the sub-carrier. It is given by [8]:

$$
R_{BOC/PRN}(\tau) = \frac{1}{2} \left( \text{tri} \left( \frac{\tau}{\tau_{R}} \right) - \text{tri} \left( \frac{\tau}{2\tau_{R}} \right) \right)
$$

(2)

As seen in (2), $R_{BOC/PRN}$ possesses two triangles perfectly situated at ±0.5 chips that can be used to remove the side peak of the BOC(n,n) autocorrelation function. There is just a sign problem for one side peak that can be taken into account when considering the use of a non-coherent discriminator. The idea is to subtract $R_{BOC/PRN}^2$ from $R_{BOC}^2$ to remove the side-peaks. A method introducing a variant of the EMLP discriminator using this technique was presented in [8]. This paper focuses on a Dot-Product (DP) discriminator that is given hereafter:

$$
D_{DP}^{BOC/PRN}(\tau_{\varepsilon}) = [(IE - IL)IP + (QE - QL)QP]_{BOC} - [(IE - IL)IP + (QE - QL)QP]_{BOC/PRN}
$$

(3)

where $IE$, $IL$, and $IP$ ($QE$, $QL$, and $QP$) represent the in-phase (quadrature-phase) early, late and prompt correlator output. The subscript $BOC$ and $BOC / PRN$ represent the BOC(n,n) autocorrelation and the BOC/PRN correlation functions.

In order to be efficiently studied, this discriminator needs to be normalized, since its output is dependent upon the signal’s amplitude. The traditional $IP^2 + QP^2$ is proposed herein. Other normalizations could be used, but it is not the intent of this paper to study the impact of different normalizations on the tracking performance. The normalized discriminator output (estimating the code phase error) has the final following form:

$$
\nu_{DP}^{BOC/PRN}(\tau_{\varepsilon}) = \frac{[(IE - IL)IP + (QE - QL)QP]_{BOC} - [(IE - IL)IP + (QE - QL)QP]_{BOC/PRN}}{(6 + d)(IP^2 + QP^2)}
$$

(4)

where $d$ is the Early-Late correlator spacing.

Fig. 2 shows the discriminator output for the proposed and traditional normalized BOC(1,1) DP discriminators for 6 and 24 MHz double-sided front-end filters, and for an early-late spacing of 0.205 chips. This normalized discriminator shows increasing response to large tracking errors, that are compensated by placing a hard limiter forcing the normalized output to be not larger than 0.4.

![Fig. 2 – Normalized Discriminator Output for BOC(1,1) Classical and Proposed Dot-Product Discriminators for a Double-Sided Front-End Filter Bandwidth of 24 (Left) and 6 (Right) MHz and $d = 0.205$ Chips](image-url)
Two major remarks can be made: (1) the stability range is slightly increased compared with the usual discriminator. It now equals 0.76 chips, compared to the usual 0.66 chips. This represents an increase of 15% of the tracking region which might be of significant importance under high noise. It has to be noted that the other unambiguous tracking techniques using a modified correlation function can have an extended tracking range up to 2 chips due to their wider correlation function. This, however, often implies a degraded resistance to thermal noise and multipath; (2) the ambiguity is removed for large front-end filter bandwidths. Indeed, the left plot in Fig. 2 shows that the discriminator output is null around the BOC(n,n) potential false lock point. However, in the narrow bandwidth case (right plot), the discriminator variation around ±0.6 chips implies a potential false lock point. The reason is that a reduced front-end filter bandwidth tends to misalign the filtered BOC(n,n) autocorrelation side-peaks and the filtered BOC/PRN correlation peaks, resulting in a potential false lock point. However, the discriminator output variations around ±0.6 chips indicates that it is fairly unstable and will not induce false tracking at usual C/N0 values (up to at least 40 dB-Hz according to the tests realized in the last section using a 1 ms coherent integration time). Yet, for higher C/N0, it could lead to a false lock. Nevertheless, this is not considered as a threat as it is easy to declare loss of lock in this case using the following test:

\[
T = (IP^2 + QP^2)_{BOC} - (IP^2 + QP^2)_{BOC/PRN}
\]  

(5)

Indeed, when looking at Fig. 3, \(T\) should have a close-to-null value around the false lock point. This combined with the fact that potential false lock can happen only at high C/N0 makes it very reliable.

![Comparison of the Correlation Functions of the Traditional and Proposed BOC(1,1) Tracking Method with a 24 MHz Double-Sided Front-End Filter](image)

Fig. 3 – Comparison of the Correlation Functions of the Traditional and Proposed BOC(1,1) Tracking Method with a 24 MHz Double-Sided Front-End Filter

The test \(T\) can also be used for acquisition purposes, as shown in [8]. It induces a degradation in acquisition of only 1 dB in terms of equivalent C/N0. It removes any chance of false acquisition on one of the side-peaks. However, it requires two complex correlators instead of one, which will result in a longer mean-time-to-first-fix when considering receivers with only a limited number of available correlators.

It is also important to note that this proposed DP discriminator has the advantage to require only a limited number of correlators, as the early-late code values can be obtained directly from the generation of the associated early-late signal waveform. As a consequence, only four complex correlators are needed (two for the BOC autocorrelation and two for the BOC/PRN correlation). This means that it uses the same number of correlators as the ‘Bump and Jump’ technique (Prompt, Early-Late, VE and VL). No extra hardware is required to implement this technique, unlike in the case of single-side band methods where extra front-end filters are needed.

Now that the principle and validity of the proposed unambiguous method have been shown, it is interesting to develop the theory of the performance of such a method, especially for the two main sources of code tracking error: thermal noise and multipath.

**CODE TRACKING ERROR OF THE PROPOSED METHOD IN WHITE NOISE**
The theoretical calculation of the error of the DP discriminator is (very) fastidious, and as a result, only the final result is shown.

Let us assume that after carrier wipe-off and correlation the correlator output is as follows:

\[
BOC_I = \sum_{i=-\infty}^{\infty} I_X(t) + n_{I_X_{BOC}} \quad \text{and} \quad BOC_Q = \sum_{i=-\infty}^{\infty} Q_X(t) + n_{Q_X_{BOC}}
\]

where \( I_X(t) \) and \( Q_X(t) \) are the unfiltered White Gaussian noise components with a constant PSD equal to \( \frac{N_0}{4} \), \( PRN(t) \) is the code modeled as a random (independent from symbol to symbol) NRZ sequence, \( SC(t) \) is the BOC(n,n) sub-carrier, \( l \) is the low-pass filter associated to the integrate and dump, \( \epsilon \) is the code delay estimation error, \( \hat{\tau} \) is the estimated code delay, and \( C \) is the received signal power.

Assuming the Dot-Product discriminator described in (3), the final code tracking error for the proposed BOC(n,n) tracking technique in thermal noise equals:

\[
\sigma^2_{\text{proposed}} = B_l d \left( \frac{C}{N_0} \frac{1 + \frac{2(4 + d)}{C N_0}}{(6 + d) T_l} \right) \text{Chips}^2
\]

where \( T_l \) is the coherent integration time and \( B_l \) is the one-sided loop filter bandwidth.

It can be noted that for \( d \ll 4 \) chips, which is the case for BOC(n,n) tracking, we have:

\[
\sigma^2_{\text{proposed}} = B_l d \left( \frac{C}{N_0} \frac{1 + \frac{4}{C N_0}}{3 T_l} \right) \text{Chips}^2
\]

For comparison, the tracking error variance in thermal noise for the traditional BOC(n,n) is given by:

\[
\sigma^2_{\text{BOC}} = B_l d \left( \frac{C}{N_0} \frac{1 + \frac{1}{C N_0}}{3 T_l} \right) \text{Chips}^2
\]

As a confirmation of the calculation, (9) follows closely the results obtained in [1] for a pure BOC tracking using an Early-Minus-Late-Power discriminator. Equations (8) and (9) show that the performances of both tracking techniques are very similar. The squaring losses are slightly higher (by a factor of 4/3) for the proposed tracking method which will

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have a noticeable impact only at very low $C/N_0$. Fig. 4 compares the theoretical code tracking standard deviation of the proposed and traditional BOC(n,n) tracking method for an integration time of 1 and 20 ms. The degradation is minimal, showing excellent behaviour of the proposed method. It is important to note that the difference between the two tracking error standard deviations is not very dependent upon the chip spacing or the coherent integration time. The loss can be quantified as less than 1 dB in terms of equivalent $C/N_0$.

**PROPOSED BOC(N,N) TRACKING METHOD MULTIPATH RESISTANCE**

The second large source of code measurement error studied herein is multipath. Fig. 5 shows the multipath envelope for the classical and proposed BOC(n,n) tracking techniques, using an early-late spacing of 0.205 chips, and a signal-to-multipath-amplitude ratio of 3 dB. Two double sided front-end filters are used: 6 and 24 MHz. The multipath envelopes corresponding to each tracking technique have approximately the same shape. The traditional BOC(n,n) tracking method has a slightly better behaviour than the proposed method for multipath delays within $[0.2;0.55]$ chips, while it is the opposite for multipath delays within $[0.55;1]$ chips. When considering the ‘Bump and Jump’ technique, the presence of multipath can significantly change the relative magnitude of the main and side-peaks, resulting in an erroneous checking process. When considering methods using a single spectral side lobe, tracking can be compared to a BPSK(n) tracking, which has a well-known larger multipath envelope. In conclusion, the proposed method offers an excellent multipath rejection comparable with the traditional BOC(n,n) tracking technique, but being unambiguous.

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SIMULATIONS

In order to test the proposed method (already tested for an EMLP discriminator in [8]), a series of simulations were done. A first test was set to confirm the unambiguous property of the proposed DP discriminator. In order to test for false lock points, two signals with fairly high $C/N_0$ (40 and 50 dB-Hz) were simulated. The proposed method uses the discriminator represented in (4). Traditional BOC(n,n) tracking uses the usual BOC(n,n) normalized DP discriminator. The signal tested is a BOC(1,1) simulated using C/A code spreading codes. The initial code delay error was set to 0.5 chips on purpose in order to simulate a false acquisition. The DLL loop bandwidth was set to 1 Hz with carrier aiding, and the integration time was set to 1 ms. A front-end filter with a double-sided bandwidth of 6 MHz was simulated. The results are given in Fig. 6. As expected, classical BOC(n,n) tracking undergoes false locks at a chip offset of 0.56 chips for the two $C/N_0$ values tested. The proposed method clearly loses lock for a $C/N_0$ of 40 dB-Hz, as predicted, and would not be used by a receiver as a measurement. For a $C/N_0$ of 50 dB-Hz, the DLL seems to lock around +0.6 chips, although its error variance appears far higher than for the classical tracking technique. This potential problem was already discussed in section II. Such an event, because it can happen only at high $C/N_0$, is very unlikely (at this $C/N_0$, the tracking should not lose lock) and it would anyway be quite simple to prevent it using the test function $T$ described in (5). Such a test was already presented in [8]. As an example, the ratio of the mean values of $P_{BOC}^2 + Q_{BOC}^2$ and $T$ is 15 for the 53 dB-Hz case, which is easily detectable.

Fig. 6 – Code Tracking Error for Traditional and Proposed BOC(n,n) Tracking Technique for $d = 0.2$ Chips, an Initial Code Delay of 0.5 Chips and a $C/N_0$ of 43 (Left) and 53 dB-Hz (Right).

The code tracking error variance in the presence of thermal noise only was also analysed through a series of tests. Seven $C/N_0$ values were tested. The loop filter settings were the same as previously mentioned. A 20-second signal was simulated to have a simulation time greater than 10 times the filter response time. The results are shown in Fig. 7.

Fig. 7 – Code Tracking Error Standard Deviation for Traditional and Proposed BOC(n,n) Tracking Technique for $d = 0.2$ Chips, an Initial Code Delay of 0.5 Chips, and $C/N_0$ values of 25, 30, 35, 40, 45, and 50 dB-Hz.
The estimated error follows the theoretical curves for high C/N₀. However, for lower C/N₀, there is a slight divergence that is probably due to the normalization (that is not considered in (7)). The difference between both methods still remains on the order of 1 dB in terms of equivalent C/N₀, which is minimal.

CONCLUSIONS

This article presents a new unambiguous method for tracking BOC(n,n) signals. This method is directly applicable to the future Galileo civil signal on L1, as it will very likely be a BOC(1,1). It has been shown that this new method offers a reliable unbiased measurement, while keeping excellent resistance to the main source of errors due to the tracking process: thermal noise and multipath. The performance of the proposed tracking scheme in thermal noise follows very closely the classical BOC(n,n) tracking performance, allowing a degradation in terms of equivalent C/N₀ of less than 1 dB. Its resistance to multipath is equivalent to the traditional BOC(n,n) natural resistance. As a consequence, this new method keeps all the advantages of the traditional BOC(n,n) tracking technique, but with the great benefit to be unambiguous. Moreover, it requires the exact same number of correlators as the ‘Bump and Jump’ technique, and does not need extra front end components, making it easy to implement in any GNSS receiver.

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